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On the dynamic stress-strain state of isotropic rectangular plates on an elastic base under vibration loads

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ABSTRACT

The problem of calculating the dynamic stress-strain state of viscoelastic rectangular isotropic plates on a deformed base, including those lying freely on the ground medium under the influence of vibration loads, is solved. Several models of the dynamic reaction of the base are considered and a qualitative comparison of the results is made. The calculations used the method of Gauss method, Muller method and least residuals.

Keywords: rectangular plate, deformed base, stress-strain state.

1. INTRODUCTION

Rectangular plates with variable geometric and mechanical parameters under dynamic vibration loads are applied in various industries and construction. A rectangular transversely dynamically loaded plate can be supported on a deformable (elastic or viscoelastic) base; for example, in the coatings of roads, bridges or runways of air fields. To study the dynamic strength and bearing capacity of such structures requires knowledge of their dynamic stress-strain state at vertical loads. The Problem of Flexural oscillations of viscoelastic plates on an elastic base is introduced; it is a topical problem of the mathematical theory of viscoelasticity. In a closed analytical form, its solution, to simplify the elastic formulation, it is possible to obtain a limited number of boundary value problems. An alternative approach to finding an approximate or semi-analytical solution to an elastic problem is to represent the solution as a series [1,2,3]. The authors [4] propose, using the variational method for elastic problems, to reduce the resolving equations to a system of ordinary differential equations. The disadvantage of these methods is their obvious dependence on the methods of setting the boundary conditions and loading patterns. In [4], a finite-difference approach is used for static loading, which in turn leads to difficulties in implementing boundary conditions. For high-order differential equations, a large-size template is used. All the above reasoning, leads to the need to develop effective methods for solving boundary value problems of the theory of plates of the operator on a deformable base (figure 1).

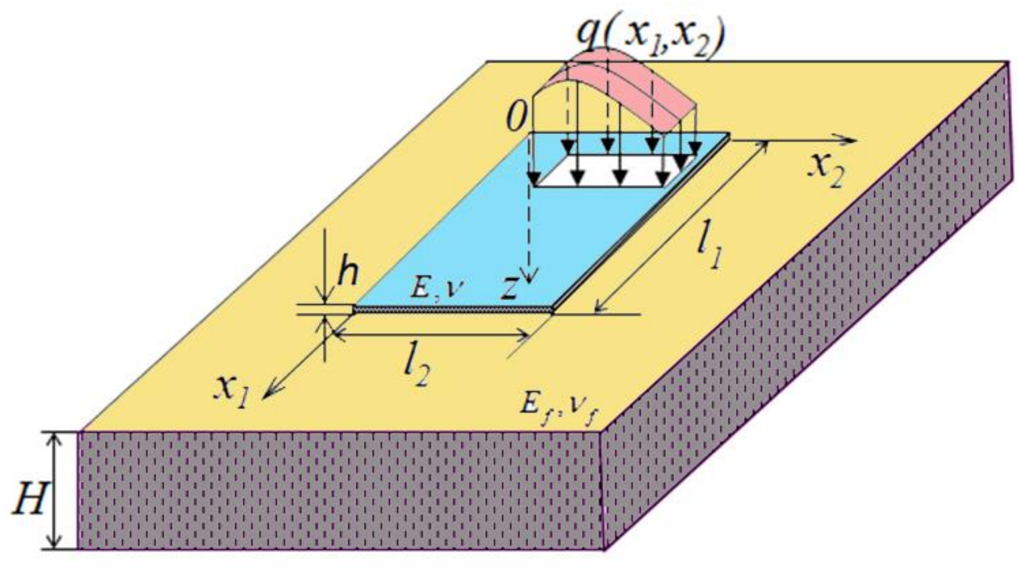

Figure 1 Plate on elastic base

2. PROBLEM STATEMENT AND SOLUTION METHODS

In this paper, the method of collocations and least residuals is used to solve the problem of plate bending numerically. The method of finite elements is well established in solving ordinary differential equations and partial differential equations for hydrodynamics problems [6]. It is used for the first time to calculate stress-strain state of plates. Consider a rectangular slab on an elastic base. The reaction of the elastic base will be considered using a one-parameter model, based on the Winkler hypothesis (hereinafter the Winkler model) [5,6,7], and two more complex two-parameter models of Vlasov [4] and Pasternak [8]. Winkler's hypothesis, suggests that the reaction of the base is proportional to the deflection of the plate

$$p = k_m \left(w - \int_0^t R(t-\tau) w(\tau) d\tau\right) \quad , \qquad (1)$$

where p - base reaction, w - plate deflection, k_m - moments coefficient of the bed (the coefficient of proportionality),determined experimentally for each type of soil. Despite its simplicity, in many cases the use of this model is sufficient to obtain acceptable results from a practical point of view. However, this representation of the soil reaction has a number of drawbacks. For example, external loads are distributed on the ground only within the area of the sole of the plate.This position does not correspond to real

observations, according to which the soil settles, and therefore is stressed outside the plate.Another disadvantage is the difficulty in determining the value of the coefficient k , which depends on the size and shape of the test stamp.A more complex model of soil reaction is laid down in two-parameter models

$$p = C_{m1}(w - \int_0^t R_1(t-\tau)w(\tau)d\tau) - C_{m2}(\Delta w - \int_0^t R_2(t-\tau)\Delta w(\tau)d\tau), \quad (2)$$

where, Δ - operator of Laplace, C_{m1}, C_{m2} - soil parameters. Here, in addition to the work of the base on the compression (Winkler hypothesis), additionally takes into account the work of the base on the shift or slice. In [4], the authors present the base as a medium in which there are no longitudinal (along the plane of the resting plate) displacements. Then the coefficients can be determined by C_{m1}, C_{m2} the following formulas

$$C_{m1} = \frac{E_0}{1-\nu_0^2}\int_0^H (\phi'(z))^2 dz, C_{m2} = \frac{E_0}{2(1+\nu_0)}\int_0^H (\phi(z))^2 dz,$$

$$E_0 = \frac{E_f}{1-\nu_f^2}, \quad \nu_0 = \frac{\nu_f}{1-\nu_f}, \quad (3)$$

E_f, ν_f - moments Young's modulus and Poisson's ratio of the elastic base, $\phi(z)$ - the function of the transverse distribution of the elastic base, which characterizes the extinction of soil tension with increasing depth H ,

$$\phi(z) = \frac{1}{2i}(e^{\gamma(z-H)} - e^{\gamma(H-z)})\sin(i\gamma H),$$

where, $\gamma = 1.5$.

In [8] it is proposed to obtain coefficients from the following considerations. Connects the intensity of the vertical resistance of the soil with its sediment, and the second independent coefficient allows you to determine the intensity of the vertical shear force. The work also describes the following possible parameters: $C_{m1} = \frac{E_0 H^{-1}}{1-2\nu_0^2}, \ C_{m2} = \frac{E_0 H}{6(1+\nu_0)}$. (4)

Let's move on to the mathematical formulation of the problem. In a rectangular area $\Omega = [0, l_1] \times [0, l_2]$ the boundary value problem describing the plate bending taking into account the elastic base reaction is considered (Figure 1) [1,4].

$$D(\Delta\Delta w(x_1, x_2, t) - \int_{-\infty}^t R(t-\tau)\Delta\Delta w(x_1, x_2, \tau)d\tau) + \rho H\frac{\partial^2 w}{\partial t^2} = q(x_1, x_2, t) - p(x_1, x_2, t),$$

where $w(x_1, x_2, t)$ -the deflection plate; $q(x_1, x_2, t)$ - external load; $p(x_1, x_2, t)$ - the reaction of the elastic foundation; $D = E_0 h^2/(12(1-\nu^2))$ - cylindrical stiffness; l_1, l_2, h - length, width, thickness of the plate; E_0, ν - moments Young's modulus and the Poisson's ratio of the plate.

The elastic base reaction is determined for each model from the corresponding formulas (1), (2) with coefficients (3) or (4). By $p(x_1, x_2) \equiv 0$ we obtain the classical equation of plate bending [1]. On the edges of the plate we will use the known boundary conditions [1]. For example, when $x_1 = 0$ maybe a free edge:

$$\left(\frac{\partial^2 w}{\partial x_1^2}+\nu\frac{\partial^2 w}{\partial x_2^2}\right)=0,\ \left(\frac{\partial^3 w}{\partial x_1^3}+(2-\nu)\frac{\partial^3 w}{\partial x_1 \partial x_2^2}\right)=Q^f.$$

Special attention should be paid to the size Q^f. This function can be interpreted as the influence of soil outside the plate on its edges [4,8]. Since the Winkler model does not account for this effect, then for it $Q^f \equiv 0$. For the two-parametric models of the edges Q^f takes the following form [4]

$$Q^{\Phi}=C_2\left(\alpha w+\frac{\partial w}{\partial x_1}-\frac{1}{2\alpha}\left(\frac{\partial^2 w}{\partial x_2^2}\right)\right),\ \alpha=\sqrt{C_{m1}/C_{m2}}\ .$$

Similarly, you can write conditions on the other edges of the plate.

Let's cover Ω the area with a rectangular grid with cells uniform in each direction Ω_i (i =1,..., N). To determine the solution in each cell, we will use the domain decomposition method-the method of iterations on subdomains (alternating Schwartz method), in which the subdomain is a cell. In each cell, you enter a local coordinate system associated with the source variables by the following formulas $y_1=(x_1-x_1^*)/h_1$, $y_2=(x_2-x_2^*)/h_2$, where $2h_1, 2h_2$ - the size of the cell in the direction x_1, x_2 respectively; (x_1^*, x_2^*) - cell center coordinate. In each cell, the approximate solution is represented as a polynomial of the fourth degree and to determine the unknown coefficients, we write a local system of linear algebraic equations . This system includes collocation equations

$$D_0\left(\frac{h_2^2}{h_1^2}\frac{\partial^4 w_i^k}{\partial y_1^4}+2\frac{\partial^4 w_i^k}{\partial y_1^2\partial y_2^2}+\frac{h_1^2}{h_2^2}\frac{\partial^4 w_i^k}{\partial y_2^4}-\int_{-\infty}^{t}R_i^k(t-\tau)\left(\frac{h_2^2}{h_1^2}\frac{\partial^4 w_i^k(\tau)}{\partial y_1^4}+\frac{\partial^4 w_i^k(\tau)}{\partial y_1^2\partial y_2^2}+\frac{h_1^2}{h_2^2}\frac{\partial^4 w_i^k(\tau)}{\partial y_2^4}\right)d\tau\right)-$$
$$-C_{m2}\left(h_2^2\frac{\partial^2 w_i^k}{\partial y_1^2}+h_1^2\frac{\partial^2 w_i^k}{\partial y_2^2}\right)+h_2^2h_1^2C_{m1}w_i^k+\rho H\frac{\partial^2 w}{\partial t^2}=h_2^2h_1^2q,$$

where w_i^k - shuffle the point точки k - the body, n - the outer normal to the border Ω_i. The boundary condition has the form in case of pinching

$$w_i^k=0,\qquad \frac{\partial w_i^k}{\partial n}=0.$$

The local system of linear algebraic equations consists of 9 integro-differential collocation equations written at the inner points of the cell. Also, at each cell boundary, depending on whether this boundary is adjacent to the boundary of the source region, three matching conditions or three boundary conditions are written. The resulting equations will be overridden. Its solution will be understood in the sense of least squares.

3. NUMERICAL RESULTS

Consider a rectangular plate on an elastic base under the action of a uniform dynamic load $q=Q_0e^{-ipt}$ [9,10] (figure 2 & 3).

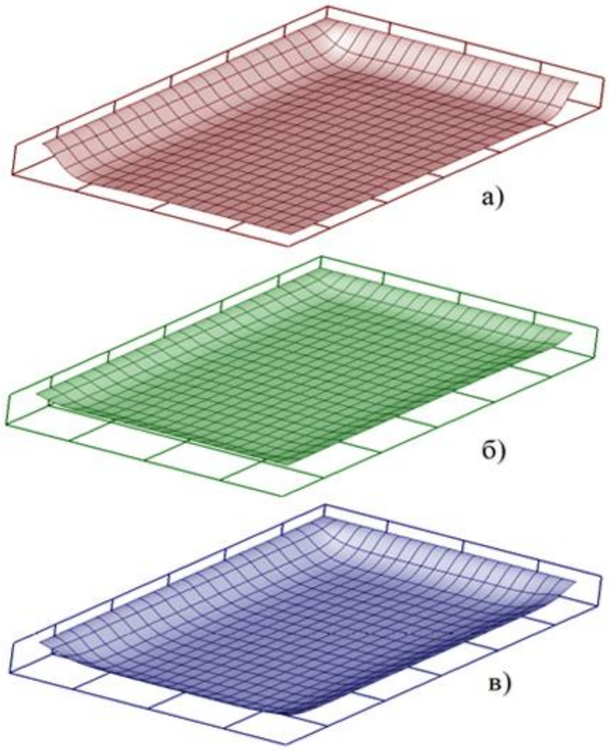

Figure 2 The amplitude Shape of a uniformly loaded plate whose two edges are pinched for Winkler (a), Vlasov (б), Pasternak (в) models.

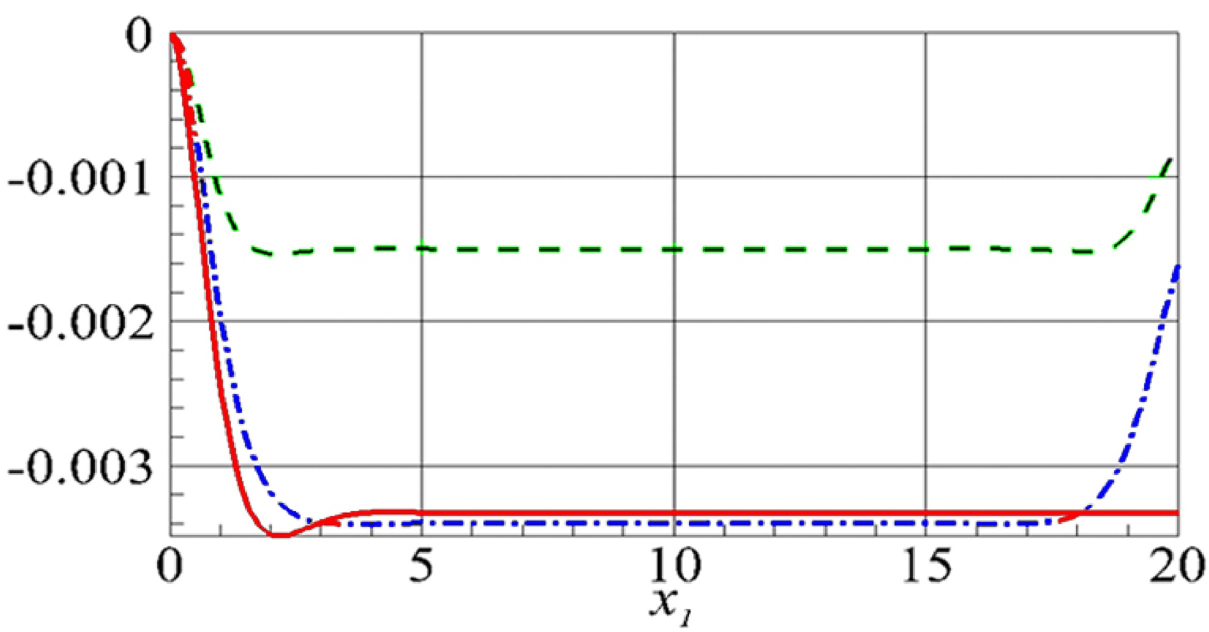

Figure 3 Section of the plate deflection amplitude at, two edges of which are pinched, for Winkler models (solid), Vlasov (dashed), Pasternak (dashed).

Two adjacent sides of the plate are pinched, the other two are free. In the experiment, the calculations for three models of the base (Fig. 2, 3) for parameters $l_1 = 2l_2 = 20$ м, $h = 0.1$ м, $H = 2$ м, E_0=200ГПа, $\nu = 0.28$, $E_f = 0.4$ ГПа, $\nu_f = 0.4$, $k = 0.3$ ГПа/м, Q_0=1 МПа . It can be seen from the figures that for two-parameter models, taking into account the function on

the free edge leads to its lifting, which from the point of view of real experience is more logical than for the case of the Winkler model, when the free edge is deformed without bending.

Consider a square plate, free-lying on the ground, modeling, for example, the Foundation of the bridge support. The plate is under the action of a uniform dynamic (harmonic) load applied to the region $[2,8]\times[2,8]$ (rice. 4, 5), $l_1 = l_2 = 10$ м, $h = 0.1$ м, $H = 2$ м, $E = 200$ ГПа, $\nu = 0.28$, $E_f = 0.4$ ГПа, $\nu_f = 0.4$, $k = 0.3$ ГПа/м, Q_0=1 МПа. In this case, the deflection of the plates does not depend qualitatively on the choice of the base reaction model, since the values of deflections on the contour are small (figure 4 & 5).

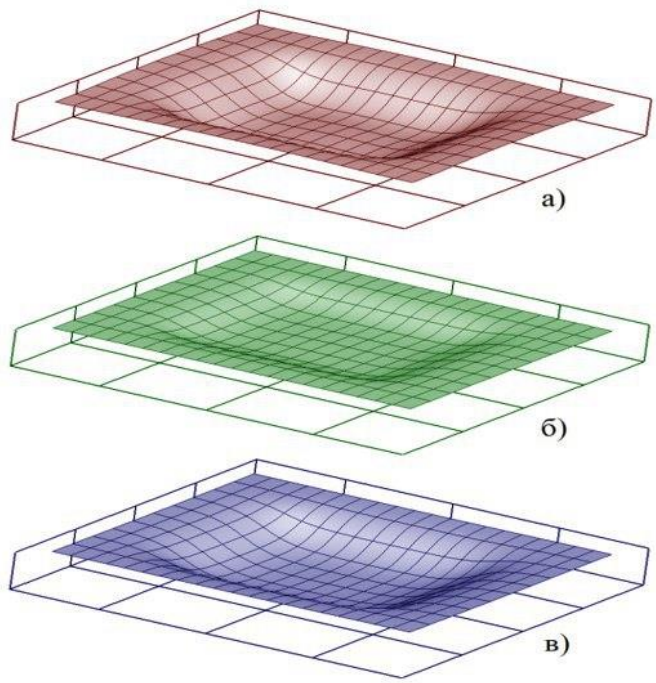

Figure 4 The amplitude Shape of the free-lying plate on the ground under a special load for the Winkler (a), Vlasov (б), Pasternak (в) models.

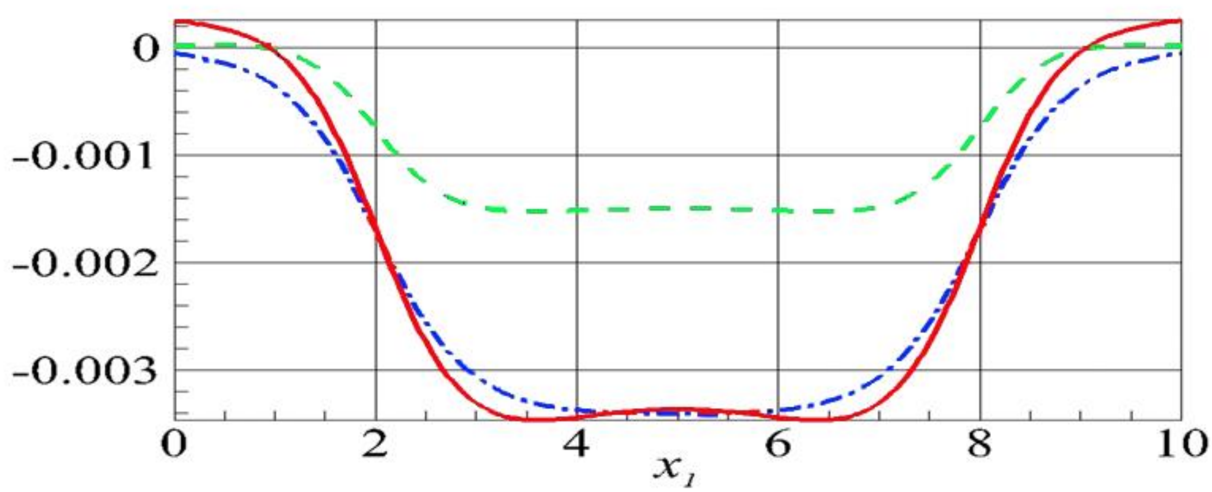

Figure 5 Section of the deflection amplitude of the free-lying plate on the ground for Winkler models (solid), Vlasov (dashed), Pasternak (dashed).

4. CALCULATIONS

Thus, an algorithm for solving dynamic and quasistatic problems is given in the paper. It was established that the models of the Winkler and Pasternak base give almost the same results, their difference is up to 5-7%, and the difference in the models of the Vlasov base is up to 30%. This can be seen from the figure. 5. It was also found that the deflection of the plates does not qualitatively depend on the choice of the base reaction model, since the deflection values on the circuit for all the models considered differ up to 10%.

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